Multi-item EOQ Calculation in Presence of Warehouse Constraints: A Simulative Analysis

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Abstract

The paper proposes an alternative method for the multi-item EOQ calculation in presence of space restrictions. The method, based on the Lagrange multipliers, consists of an iterative procedure which analyses dynamically, through the simulation, the effect of consumption rates and delivery times, determining the real stock overlapping in a fixed time period. This makes possible a more effective exploitation of the warehouse space that brings the lot sizes nearer to their optimal values. The proposed method is targeted to surmount the limits of classical approaches: firstly, the procedure calculates the lot sizes with a dynamic approach based on the simulation of the total stock behaviour in a fixed time horizon, with the aim of maximizing the space saturation; subsequently, the so obtained lot sizes are corrected to reduce further the total management cost. In particular, this correction is based on the analysis of the slope of the cost function of each item. In order to obtain it, three alternative variations of the dynamic procedure are considered. The three alternatives have been compared among them and with the traditional Lagrange method in different scenarios, showing interesting results in terms of reduction of the total management cost.

Keywords
Inventory Management, Constrained EOQ, Lagrange Multipliers, Dynamic Approach

1. Introduction and Literary Review

The Economic Order Quantity (EOQ) problem is a fundamental problem in supply and inventory management. In its classical setting, solutions are not affected by the warehouse capacity. When the warehouse space is not enough the problem has to be solved under this constraint. Various approaches can be found in literature concerning the analysis of the multi-item inventory problem with a capacity constraint on a single resource. Among the classical approaches to the solution of the constrained EOQ calculation, the most widespread is based on the application of the Lagrange multipliers method. With this approach the objective is to assure that the total available capacity is not exceeded when the various products, with independent cycle times, will eventually reach a simultaneous peak of stock. The planning problem is therefore formulated as a minimization problem with a single constraint and Lagrange multipliers are used to solve it (Hadley and Whitin 1963, Parsons 1966, Phillips et al. 1973). Another approach is based on the fixed-cycle approach. This procedure assumes a fixed cycle time for all items; once the cycle is obtained, the orders for the n items are phased within the cycle. Phasing is used to avoid situations where peaks of stock are reached simultaneously. In this case, the problem is to decide on the joint ordering cycle and the phasing within the cycle, so as to minimize total cost while satisfying the capacity constraint throughout the cycle. This approach can be found in the works of Parsons (1966), Goyal (1976), Zoller (1977) and many others. The two classical methods have been compared by Rosenblatt (1981).

Also a third approach has been largely treated in literature; it is based on the use of individual cycle times which are integer multipliers of a basic cycle time. Starting from this idea, many solution procedures were developed, also for the unconstrained problem, where the economic advantage of joint replenishment can be realized. Goyal and Belton (1979), Kaspi and Rosenblatt (1983) and Roundy (1985) propose solutions and applications to this method. A comparative study of the three above mentioned methodologies is provided by Rosenblatt (1985). These methodologies generate stationary ordering policies, which result not completely satisfactory because refer to a resource, the warehouse, which has a strongly dynamic nature. In fact the available capacity of the warehouse, due to its strict connection with the production activities, is continuously variable and, for this reason, it cannot be...
considered as a static constraint. This necessity has been already considered in literature: Hall (1988) and Rosenblatt and Rothblum (1990) consider the capacity of the system as a decision variable and not as given data. In particular, extra warehouse space can be purchased or rented out if needed, in order to obtain an ordering cost function more general than functions previously considered in the literature. This approach provides more flexibility for the decision makers but requires a more extensive computational effort, which makes more difficult its implementation in a real context. Other authors propose non stationary approaches able to guarantee cheaper solutions but also in this case the complexity of the optimizing algorithms make them not very effective in real contexts (Guder et al. 1995).

Recent studies on multi-item EOQ policies have been generally targeted to propose alternative methods to the classical derivation of the total cost function (Mondal and Maiti 2003, Chang et al. 2005, Minner 2007) but in all these cases the effect of the resource constraint introduction is no more investigated. The problem of the constrained multi-item EOQ has been resumed by Iannone et al (2010) which propose an alternative method, based on the Lagrange multipliers, that considers the space resource as a changeable limit, depending on the real level of stocks at every time. The method consists of an iterative procedure which, after a first calculation of the constrained EOQs with the traditional Lagrangian method, analyses dynamically, through the simulation, the effect of different consumption rates and delivery times, and determines the real overlapping of the stocks in a fixed period of time. This in order to correct the lot sizes at each order launch. In this paper, the method proposed by Iannone et al. (2010) has been developed in three different variations of the lot sizes calculation criterion and compared, in terms of material management costs, with the traditional Lagrange technique, in different scenarios.

2. The dynamic approach to the constrained lot sizes calculation

2.1 Lot sizes dynamic calculation

As specified by Iannone et al (2010), the main difference of the dynamic approach with the traditional Lagrange method is the introduction of the time variable \( t \) into the space constraint formula:

\[
V(t) = \sum_{i=1}^{N} v_i q_i(t) \leq V_{TOT}, \forall t \leq T
\]

(1)

where \( N \) is the number of different items managed in the warehouse, \( v_i \) is the specific volume of the \( i \)th item, \( q_i(t) \) is its quantity in stock at the time \( t \) and \( T \) is the time horizon during which the constraint has to be satisfied.

The lot sizes \( Q_i \) are obtained through an iterative procedure in which \( \lambda \) is initially set to 0 and progressively increased. For each increase of \( \lambda \) the lot dimensions \( Q_i \) are calculated through the Lagrange classical formula:

\[
Q_i = \frac{d_i \cdot C_{i,a} \cdot T}{k_i \cdot \left( \frac{T}{2} + \lambda \cdot v_i \right)}
\]

(2)

where \( d_i \) is the daily demand for the \( i \)th item, \( C_{i,a} \) is the cost of order launch for the \( i \)th item and \( k_i \) is the holding cost per unit of time of the \( i \)th item.

Once calculated the lot sizes \( Q_i \), the level \( V(t) \) of the total stock, variable with \( t \), is simulated over the whole time horizon \( T \). If the constraint is not respected, \( \lambda \) is increased and the simulation restarts. The exit condition of the iterative loop is the respect of (1). The logic of the algorithm is schematized in Figure 1.

In this approach the length of the time horizon assumes great relevance. In order to saturate better the warehouse capacity, the simulations start from the second day with the hypothesis that in the first day the quantities in stock correspond to the lot sizes calculated through the Lagrange multipliers method. These quantities saturate completely the available space. In the following days, the stock levels will be surely lower because of the consumption due to the items daily demands. After the first day, the new lots will be ordered, when necessary, according to the quantities calculated with the dynamic method. The final solution is characterized by a value of \( \lambda \) which can be lower than that one calculated through the Lagrange method because the volume actually occupied by the stocks is normally smaller than that necessary for all the lots in the initial condition.
2.2 Lot sizes correction
As the Lagrange method, also the above described dynamic calculation procedure gives the same importance to all the items. In both cases in fact, the lot sizes are reduced proportionally to the relative space occupation, without considering the slope of the cost function of each item. For this reason, the combination of lot-sizes obtained through the Lagrange method can lead also far to the minimum of the total management cost function. Figure 2 exemplifies the effect of the same reduction of the lot dimension on two items with different cost functions: it is easy to understand that the space should be saved with a more marked reduction of the item 2 without change the lot of item 1. Starting from these considerations the opportunity to define a correction of the formula for the lot calculation appears to be evident. In particular, the correction consists of the addition of a term which gives a more marked reduction of the initial value of the lot-size to the items with a slighter slope of the cost function, while the items with steeper slope of the function sustain a more limited reduction of their initial lot-sizes. With this corrective term the formula for the calculus of the lot sizes become the following:

\[
Q^*_{i} = \frac{d_i \cdot C_{ij} \cdot T}{k_i \cdot T + \lambda \cdot v_i + f\left(\frac{dC_i}{dQ_i}, \mu\right)}
\]  

(3)
Figure 2: Effect of equal reduction of lots with different cost functions

2.3 Linear and logarithmic variations
In Iannone et al. (2010) two were the variations (linear and logarithmic) of the proposed approach defined in order to reduce further the total cost of stock. They specify differently the correction term in the denominator of (3). In the Linear variation it was:

\[ f\left(\frac{dC_i}{dQ}, \mu\right) = \tan \mu \ast \left(\frac{dC_i}{dQ} - \frac{\overline{dC}}{\overline{dQ}}\right) \approx \mu \ast \left(\frac{dC_i}{dQ} - \frac{\overline{dC}}{\overline{dQ}}\right) \] (4)

where \( \overline{dC} \) is the mean of the derivatives of the N items.

In this paper we distinguish two linear variations, according to the typology of mean specified in (4): in the first case the mean is arithmetic and given by:

\[ \overline{dC} = \frac{\sum_{i=1}^{N} dC_i}{N} \] (5)

while in the second case the mean is harmonic and is given by:

\[ \overline{dC} = \frac{N}{\sum_{i=1}^{N} \frac{1}{dC_i}} \] (6)

In both cases the lot sizes can be calculated as following:

\[ Q^{*} = \sqrt{\frac{2 \cdot C_i \cdot d_i \cdot T}{k_i \cdot T + 2 \cdot \lambda \cdot v_i + 2 \cdot \mu \cdot \left(\frac{dC_i}{dQ} - \frac{\overline{dC}}{\overline{dQ}}\right)}} \] (7)

In the Logarithmic variation the corrective term has the following expression:

\[ f\left(\frac{dC_i}{dQ}, \mu\right) = \mu \ast \ln \left(\frac{dC_i}{dQ} - \ln \frac{\overline{dC}}{\overline{dQ}}\right) = \mu \ast \ln \left(\frac{dC_i}{\overline{dC}}\right) \] (8)
where \( \frac{dC}{dQ} \) is the harmonic mean, given by (6). The lot sizes formula is the following:

\[
Q_i^{**} = \frac{2 \cdot C_i \cdot d_i \cdot T}{k_i \cdot T + 2 \cdot \lambda \cdot v_i + 2 \cdot \mu \cdot \ln \left( \frac{dC}{dQ} \right)}
\]

As already specified by Lannone et al. (2010) the existence domain of the \( \mu \) parameter is identified through the respect of the following inequalities:

\[
\mu > \max_{i=1..N} \left\{ \frac{k_i \cdot T + 2 \cdot \lambda \cdot v_i}{2 \cdot \left( dC_i \cdot \frac{dC}{dQ} - dC_i \right)} \right\}
\]

for the two linear variations and:

\[
\mu > \max_{i=1..N} \left\{ \frac{k_i \cdot T + 2 \cdot \lambda \cdot v_i}{2 \cdot \ln \left( \frac{dC_i}{dQ} - dC_i \right)} \right\}
\]

for the logarithmic variation. These conditions respect the obvious necessity to have real values of \( Q_i^{**} \) in (7) an (9) formulas.

### 2.4 The algorithm of lot sizes optimization

Figure 3 shows the complete flow chart of the proposed algorithm for the lot size optimization through the application of the before described variations to the dynamic calculation procedure.

The first step of the algorithm is the initialization of \( \lambda \) \( \in \mu \), with the subsequent running of the two cycles: once \( \mu \) is fixed all the values of \( \lambda \), starting from 0, will be tested. \( \lambda = 0 \) represents the condition of absence of space constraint. The minimum value of \( \mu \) is analytically calculated through the (x) and (y) formulas.

After the initialization of \( \mu \) and \( \lambda \) it is possible to start with the lots calculation. This requires the values of the derivatives, which, in their turns, require the values of \( Q \). In order to surmount this typical problem of circularity, the first values are taken from the Lagrange dynamic procedure. With these values, the derivatives and their means are calculated and the new lot sizes can be obtained through the application of one of the three variations.

In order to do that, with the \( \mu \) parameter initially fixed to its minimum value, the complete simulation of the stock levels is performed, with the verification of the space constraint satisfaction for every day of the time horizon. Each violation of this constraint imposes the increment of \( \lambda \), and a restarting of the simulation. When the loop is concluded, the total costs are calculated and compared with those ones obtained in the Lagrange dynamic calculation phase, which represent the best solution so far.

If the cost condition \( (C_{tot} < C_{min}) \) is verified, the values of \( Q_i \), \( C_{tot} \), \( \lambda \) and \( \mu \) are memorized as new best solution; then \( \mu \) is increased and the loop restart from the beginning. When this condition is violated, the procedure ends and furnishes in output the lot sizes corresponding to the minimum costs, together with the relative values of \( \lambda \) and \( \mu \).

This is assured by the experimental analysis of the total cost function which always presents a minimum value when \( \lambda \) is progressively increased, as shown in Figure 4.

The procedure provides for the possibility to fix a maximum value to both \( l \) and \( m \) in order to limit the number of iterations before its ending. This can sometimes reduce the run duration of the algorithm but do not assure the reaching of the best solution. Another choice with a great influence on the computational time of the procedure is the value of the \( \varepsilon \), which increases \( \lambda \) and/or \( \mu \) at every iteration: it is easy to understand that small values of \( \varepsilon \) require longer running times but let to better solutions.

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Figure 3: Flow chart of lot sizes optimization algorithm

Figure 4: Graph of the Total Cost function of the $\mu$ parameter
3. Experimentation

The experimentation phase has been carried out with the two following purposes:

1. To analyze the behavior of the approach with its three variations when the external conditions change. In particular, the parameters that have been modified are the time horizon of the simulation, the number of items contemporarily managed into the warehouse and the available space of the warehouse.

2. To establish, relatively to the tested range of parameters, which one of the three variations is able to furnish the best performances in terms of total costs.

3.1 Objective Function definition

In order to pursue the above defined aims it has been necessary to define an objective function which make possible to compare the cost performances of different solutions. This because different configurations are subjected to different values of total cost, also in the optimal conditions and, therefore, a simple comparison of the relative costs is not significant for any kind of consideration about the performance of the variations.

To surmount this limit the objective function has been defined as following:

$$F_{obj} = \frac{\Delta C_{TOT}}{\Delta C_{EOQ}} = \frac{C_{Lagrange} - C_{Variation}}{C_{Lagrange} - C_{EOQ}}$$

where: $C_{Lagrange}$ is the total cost relative to the application of the traditional Lagrange method, $C_{variation}$ is the total cost relative to the application of the specific variation of the dynamic approach and $C_{EOQ}$ is the total cost relative to the application of the EOQ method, without any limitation on the space availability. Of course, this last case leads to the minimum cost and for this reason its distance to the result of the application of the Lagrange method represents the benchmark for the variations performances. This concept can by clarified by Figure 5, in which the three levels of cost are graphically represented.

![Figure 5: Levels of cost for the definition of the objective function](image)

Is clear that $0 \leq F_{obj} \leq 1$. In particular, if $F_{obj} = 0$, the variation has the same cost of the Lagrange method; on the contrary, if $F_{obj} = 1$, its performance is equal to the EOQ and so it is the best possible. In general, the higher is its value, the better is the performance of the variation. By means of the so defined function also different configurations (i.e. different numbers of items and/or warehouse capacities) can be compared between them.

3.2 Test planning

For the experimental tests targeted to give a first answer to the two above mentioned questions, a set of 8 different items has been considered. For them, all the characteristics necessary for the analysis have been randomly chosen in a range initially fixed. These characteristics and the relative ranges of values are reported in Table 1.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Measure Unit</th>
<th>Range</th>
<th>Item 1</th>
<th>Item 2</th>
<th>Item 3</th>
<th>Item 4</th>
<th>Item 5</th>
<th>Item 6</th>
<th>Item 7</th>
<th>Item 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daily Demand ($d_i$)</td>
<td>Units/day</td>
<td>1÷50</td>
<td>14</td>
<td>30</td>
<td>23</td>
<td>34</td>
<td>17</td>
<td>15</td>
<td>12</td>
<td>30</td>
</tr>
<tr>
<td>Ordering Cost ($C_i$)</td>
<td>€/100</td>
<td>20÷100</td>
<td>56</td>
<td>38</td>
<td>28</td>
<td>25</td>
<td>50</td>
<td>30</td>
<td>50</td>
<td>85</td>
</tr>
<tr>
<td>Holding cost ($k_i$)</td>
<td>€/(unit*day)</td>
<td>0,001÷0,05</td>
<td>0,008</td>
<td>0,004</td>
<td>0,0010</td>
<td>0,009</td>
<td>0,008</td>
<td>0,005</td>
<td>0,005</td>
<td>0,02</td>
</tr>
<tr>
<td>Specific volume ($v_i$)</td>
<td>m$^3$/unit</td>
<td>0,1÷10</td>
<td>0,860</td>
<td>6,570</td>
<td>5,890</td>
<td>3,21</td>
<td>8,25</td>
<td>6,34</td>
<td>7,72</td>
<td>0,4</td>
</tr>
<tr>
<td>Purchasing cost ($P_i$)</td>
<td>€/unit</td>
<td>1÷30</td>
<td>3,83</td>
<td>2,70</td>
<td>5,20</td>
<td>25,5</td>
<td>8</td>
<td>9</td>
<td>8,1</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 1: Randomly fixed items characteristics
The performance of all the three defined variations (Lin = Linear with arithmetic mean, Lin2 = Linear with harmonic mean and Log = Logarithmic) have been measured in different scenarios obtained by changing the following parameters:

1. \(N = \) Number of Items; the chosen values are 4, 6, 8. In particular the maximum value has been fixed to 8 in order to reduce, in this preliminary phase, the computational time of the tests.

2. \( t = \) Time horizon; for the time parameter, the chosen values are \( \frac{1}{4}T, \frac{1}{2}T \) and \( \frac{3}{4}T \), where \( T \) is defined as the time in which all the lots, calculated with the Lagrange formula, will be contemporarily present in the warehouse after the initial instant. This value is the least common multiple (lcm) of the consumption times of the items, obtainable from the following formula:

\[
T = \text{lcm} \left\{ \frac{Q_i}{d_i} \right\}
\]

With the values reported in Table 1 it is easy to verify that \( T \approx 780 \) days, so the three levels correspond to 195, 390 and 585 days respectively.

3. \( V_{\text{tot}} = \) Warehouse capacity; the chosen values are 2000 m\(^3\), 3000 m\(^3\) and 4000 m\(^3\). These values are included in a range of 10\%÷30\% of the total space required by the EOQs. In this phase we have preferred a strong reduction of the available space in order to have an adequate distance between the solution corresponding to the EOQ and that one coming from the Lagrange method application. In this way the performances of the different variations can be evaluated with more accuracy.

Table 2 shows the matrix of the trials consequent to the choices made for the parameters object of investigation. The combinations of the 3 parameters with 3 level each one are 27. Since all the combinations are to be tested with the 3 variations of the method, the total number of simulations to perform amounts to 81.

### Table 2: Matrix of trial of the experimentation

<table>
<thead>
<tr>
<th>No. of Items</th>
<th>Warehouse Capacity</th>
<th>Time horizon</th>
<th>(\frac{1}{4}T)</th>
<th>(\frac{1}{2}T)</th>
<th>(\frac{3}{4}T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2000 m(^3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3000 m(^3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4000 m(^3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2000 m(^3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3000 m(^3)</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>4000 m(^3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>2000 m(^3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3000 m(^3)</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>4000 m(^3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### 3.2 Test execution and results

The simulation program has been realized in MATLAB. The 81 simulations have been carried out with single repetition modality because of the deterministic nature of the so structured problem and the results, in terms of mean value of the objective function, are reported in Table 3.

### Table 3: Results of the simulations

<table>
<thead>
<tr>
<th>N</th>
<th>Vtot</th>
<th>t</th>
<th>Lin</th>
<th>Lin2</th>
<th>Log</th>
<th>N</th>
<th>Vtot</th>
<th>t</th>
<th>Lin</th>
<th>Lin2</th>
<th>Log</th>
<th>N</th>
<th>Vtot</th>
<th>t</th>
<th>Lin</th>
<th>Lin2</th>
<th>Log</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>1/4 T</td>
<td>0.25649</td>
<td>0.26623</td>
<td>0.25974</td>
<td></td>
<td>2000</td>
<td>1/4 T</td>
<td>0.23077</td>
<td>0.23077</td>
<td>0.24821</td>
<td></td>
<td>2000</td>
<td>1/4 T</td>
<td>0.27746</td>
<td>0.25109</td>
<td>0.25235</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1/2 T</td>
<td>0.22042</td>
<td>0.22852</td>
<td>0.2677</td>
<td></td>
<td></td>
<td>1/2 T</td>
<td>0.20492</td>
<td>0.21311</td>
<td>0.23975</td>
<td></td>
<td></td>
<td>1/2 T</td>
<td>0.20711</td>
<td>0.21966</td>
<td>0.23222</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3/4 T</td>
<td>0.2029</td>
<td>0.22847</td>
<td>0.18172</td>
<td></td>
<td></td>
<td>3/4 T</td>
<td>0.20471</td>
<td>0.19959</td>
<td>0.23</td>
<td></td>
<td></td>
<td>3/4 T</td>
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<td></td>
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<tr>
<td>3000</td>
<td>1/4 T</td>
<td>0.23125</td>
<td>0.23125</td>
<td>0.24375</td>
<td></td>
<td>3000</td>
<td>1/4 T</td>
<td>0.25623</td>
<td>0.25623</td>
<td>0.2758</td>
<td></td>
<td>3000</td>
<td>1/4 T</td>
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<td>0.30794</td>
<td>0.32511</td>
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<tr>
<td></td>
<td>1/2 T</td>
<td>0.23171</td>
<td>0.23171</td>
<td>0.24451</td>
<td></td>
<td></td>
<td>1/2 T</td>
<td>0.248</td>
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<td>0.26311</td>
<td></td>
<td></td>
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<td>0.26838</td>
<td>0.27375</td>
<td>0.22857</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3/4 T</td>
<td>0.21503</td>
<td>0.21503</td>
<td>0.24008</td>
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<td></td>
<td>3/4 T</td>
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<td>0.24852</td>
<td></td>
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<td>0.27857</td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>1/4 T</td>
<td>0.19565</td>
<td>0.19565</td>
<td>0.20652</td>
<td></td>
<td>4000</td>
<td>1/4 T</td>
<td>0.28117</td>
<td>0.30367</td>
<td>0.34254</td>
<td></td>
<td>4000</td>
<td>1/4 T</td>
<td>0.32247</td>
<td>0.32247</td>
<td>0.32899</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1/2 T</td>
<td>0.18033</td>
<td>0.18033</td>
<td>0.20219</td>
<td></td>
<td></td>
<td>1/2 T</td>
<td>0.28173</td>
<td>0.29554</td>
<td>0.31764</td>
<td></td>
<td></td>
<td>1/2 T</td>
<td>0.27524</td>
<td>0.27524</td>
<td>0.32411</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3/4 T</td>
<td>0.18545</td>
<td>0.18545</td>
<td>0.2</td>
<td></td>
<td></td>
<td>3/4 T</td>
<td>0.26427</td>
<td>0.24862</td>
<td>0.28176</td>
<td></td>
<td></td>
<td>3/4 T</td>
<td>0.31522</td>
<td>0.27717</td>
<td>0.27717</td>
<td></td>
</tr>
</tbody>
</table>
Figure 6 summarizes the mean performances of the three variations as functions of the time horizon, the number of items and the warehouse capacity. As expected, the trend of all the performances is decreasing with the time horizon, due to the even stronger effect of the constraint. In fact with a longer time horizon, the probability to have a peak of stock increases and, consequently, the lot dimensions get nearer to the values calculated with the Lagrange method. In particular, if \( t = T \), where \( T \) is given by (13), the procedure should restore the same results of Lagrange. On the contrary, the increasing trends of the performances with the number of items and with the available space are not easily predictable and therefore appear more interesting. A possible explanation is that when the problem leaves a wider margin of action in the search of the optimal solution, the procedure is able to improve more significantly the initial solutions. Of course a higher number of items and more space in the warehouse should increase that margin. For this phenomena, however, a deepen investigation seems to be necessary before drawing conclusions of general validity.

![Figure 6: Mean values of \( F_{obj} \) with different Time horizons (t), Numbers of items (N) and Warehouse capacities (V_{tot})](image)

Another important outcome of this test is the best performance of the logarithmic variation in most of the scenarios. In fact, it has produced the highest value of the objective function in 22 out of the 27 investigated combinations of parameters, versus 2 out of 27 obtained by each one of the two linear variations. This could lead to consider the logarithmic as the best variation of the proposed method and, as a consequence, to exclude the two linear variations, but also in this case further verifications are planned in order to strengthen or deny this important conclusion.

4. Conclusions
In this paper the dynamic approach to the lot sizes calculation in presence of space constraints proposed by Iannone et al. (2010) is drawn on, improved with a more efficient optimization procedure and with the addition of a third variation of the lot sizes correction mechanism and finally tested in simulated scenarios. The tests, carried out by varying the parameters with a marked effect on the analysis, have been targeted to furnish initial information about the general validity of the methodology and to understand which variation, among the proposed ones, is able to guarantee the best performances. About the general validity of the approach, the tests evidence that it allows to obtain cost savings generally included between 20% and 30% of the value lost with the introduction of the space constraint. As regards the comparison among the variations, the logarithmic has produced the highest performances and for this reasons it could be considered the best but it is also necessary to understand if it is really like that or simply there are convenience domains of the three variations; in this latter case, they should be also defined. Coherently with these considerations, further studies will be initially oriented to enlarge the space of investigation for the performance tests. This in order to made more robust the conclusion of this first analysis. After that, the subsequent purpose it to compare the set up algorithm not only with the classical Lagrange technique but also with other methodologies available in the literature in order to verify its ability to find optimal solutions to the multi-item constrained EOQ problem.

References


