Project Price Modeling by Optimal Fixed Price Incentive Contract

Kamrul Ahsan
School of Management and Information Systems
Victoria University
Melbourne, Australia

Hiroaki Matsukawa
Department of Administrative Engineering
Keio University, Japan

Abstract

The project fixed price incentive (FPI) contract imposes either penalty or incentive for cost excess or cost savings. However, problems arise with target price and refunding rate of the project, which are determined by negotiation and often require compromise with an unavoidable, unfair contract. In this paper we propose an analytical model that can resolve the unfair contract dealing of FPI. Our objective is to find an optimal solution to maximize total profit and to determine a target price in the FPI approach. We use cooperative game theory and compare results with Firm Fixed Price (FFP) approach to demonstrate the significance of using the proposed model for price negotiation.

Keywords
Project contract; incentive contract; fixed price incentive (FPI); risk management

1. Introduction

The dimensions of project priorities and constraints change day by day. These days cost reduction, contract negotiation and effective business communication are becoming bigger issues for projects. With the increasing uncertainty in global business, advancement of technology and increased customer demand there is future promise in the project management area. On the project site, there are various changes everyday. Project life cycles are shorter than planned and uncertainties are more than expected, and many projects fail because they can’t cope with change and uncertainties.

To adjust with market demand and uncertainties, advancement in project management tools and techniques started from 1917, when Henry Gantt developed the famous Gantt chart. Further significant advancement in this area occurred in 1950, with the introduction of PERT (Program Evaluation and Review Technique) in missile development and CPM (Critical Path Method) at Du Pont. The project progress measuring tool, Earned Value Management System (EVMS) was the next significant progress in 1962. Further, with the development of latest computer technology, many project scheduling techniques are being implemented to solve real life projects.

Although there has been remarkable growth in the areas of project scheduling and planning procedure, less attention has been given to project contract and negotiation. A project management literature survey amongst the top management journals finds that less research has been conducted in the discipline related to engineering and construction, contracts, expert witness, and their legal (EC/Contract/Legal) implications. Among the disciplines, research focus on EC/Contract/Legal is only 3% and the research area is struggling for a position in the journals (Kwak and Anbari 2008).

A project contract is an outline that clarifies or specifies possible project risks and their allocation, sufficient rewards and incentives for the contractors, sanctions for negligence and an efficient resolution of disputes between client and contractor. Jensen (2000) (cited in Muller and Turner 2005), defines contract as an agency relationship under which one party(the principal) engages another(the agent) to perform some service on their behalf which involves delegating some decision making authority to the agent. The appropriate form of contract is necessary to motivate the contractor to reduce the cost of works and achieve the client’s objectives. The contract methods should
aim to align the contractors’ objectives with the owners or clients, by providing appropriate incentives through a
win-win game (Turner 2004). Within the contract agreement stage, both parties try to promote their interests by
introducing contract conditions that protect their final goals. In this respect, one of the most important elements will
be the choice of contract to be used (Veld and Peters 1989). The appropriate form usually depends on several issues:
(1) who controls the risk (2) nature of the project, and (3) location of the uncertainty (Turner, 2004).

In project management literature, two categories of contract research can be found that deal with soft and hard
contract issues. Soft issues are mainly focused on contract negotiation (Koskinen and Mäkinen 2009),
communication issues between project owner and manager (Müller and Turner, 2005), trust and contracting relation
in contract administration (Zaghloul and Hartman 2003), and perceptions of owners and contractors concerning
incentive/disincentive contracting (Bubshait 2003). Among hard issues, research on risk sharing techniques between
client and contractor (Medda 2007; Broome and Perry 2002; Berends 2000; Al-Subhi Al-Harbi 1998; Samuelson
1986), determination of final cost of construction at bid time (Nutakor 2007), minimum contract bid modeling for
incentive and disincentive bidding (Shr and Chen 2003), development of contact payment terms (Ward and
Chapman 1994), and model for contract payment or contract profit (Ward and Chapman 1995) are remarkable.

The fixed price and cost plus are two basic types of contracts used in project management. If the project process
is uncertain, but the project product is certain, fixed price contracts are preferred. If both product and process are
uncertain, cost plus contracts, based on an alliance arrangement are preferred (Turner and Simister 2001). The fixed
price and cost plus contracts have been extensively discussed in economics (Bajari and Tadelis 2001; Hayden and
Boldue, 1989; McAfee and McMillan, 1986) and business literature (Paul and Gutierrez 2005; Turner and Simister
2001; Connor and Hopkins 1997; Ward and Chapman 1995; Ward and Chapman 1994; Paliwoda and Bonaccorsi
1994; Samuelson 1986; Ryan et al. 1986; Meinhart and Delionback 1968).

Within fixed price contracts the Fixed Price Incentive (FPI) contract has more practical application in terms of
risk sharing between the client and contractor. FPI contract, features a fixed ceiling cost above which the contractor
must assume full responsibility for loss (Meinhart and Delionback 1968). One noteworthy study for the FPI contract
in projects was conducted by Ward and Chapman (1995). To evaluate different forms of FPI contracts, the study
develops a triangular probability density function of contract payment and contractor’s profit model. However, FPI
contracts do not appear as widely as they might. Possible reasons for unavailability of FPI research may be lack of
familiarity with the concept, or uncertainty about how to devise suitable forms of incentives for different situations.
In FPI it is difficult to theoretically determine optimal price of the contract. In the absence of any theoretical
methodology to determine optimal contract, clients and contractors do manage to negotiate a mutually acceptable
FPI contract. Additionally, there may be concerns that incentive contracts are difficult and time consuming to

The game theory approach extensively applies to the analysis of multi-player decision problems, where the
players behave in a conflicting or cooperative situation, to seek optimal solution (Leng and Zhu 2009). Because of
the undeceiving nature of contract profit shares and the win-win nature of the modern project contract objective,
cooperative game theory can be used as a good tool to allocate the profits among the project contract players. For
optimal project contract modeling, application of game theory is scarce. Five decades ago Meinhart and Delionback
(1968) examined contractor role in research and development projects when operating under incentive contract. The
authors developed a conventional minimax approach of game theory for aiding decisions in cost plus incentive fees
(CPIF) contracts. Recently, Medda (2007) proposes a model under final offer arbitration game framework for
allocating risks between private and public sectors in transport infrastructure. In the proposed model, the agents
compete to achieve the most reasonable offer in the settlement.

Unfortunately, less research has been conducted on FPI contract and there has been less of a focus on applying
game theoretic approaches. To address the research gap in FPI contract, we propose a model that fixes the optimal
price for FPI by maximizing the profit for both client and contractor. We use cooperative game theory to solve the
problem and compare the result with firm fixed price (FFP) contract. To the best of our knowledge, this is the first
try to analyse FPI contract price with a co-operative game theoretic approach where both players’ interest is
given priority. The outline of the paper is as follows. In section two we describe different types of project contracts
and focus on incentive contracts and recent developments of contracts in project management literature. Further in
section three, we explain the proposed FPI model. Finally in our conclusion we provide a summary of the findings.

2. Project Contracts

Contract is a promise or set of promises between parties, it is an instrument for balancing risks, with the distribution
of risks impacting the motivation of contractual partners to manage the risks towards project success (Dingle and
Topping 1995). A well-drafted contract is necessary for a satisfactory project outcome for both client and contractor.
Often problems occur due to adverse client-contractor relationship caused by moral hazard, adverse selection and risk sharing (Ward and Chapman 1994). Amongst the problems of risk sharing, a satisfactory negotiation is essential for an appropriate allocation of risk between the involved parties. The consequences of the problem may be controlled through the introduction of risk management strategies and incentive systems within the terms of contract.

2.1 Major Project Contracts

When developing a project contract strategy, the client should choose a contract type that develops an appropriate cooperative relationship between themselves and their contractors, and provides incentives to motivate the contractors to achieve their objectives (Turner 2004). Project contracts are incomplete; a flexible farsighted governance structure is required for every project contract (Müller and Turner 2005). The selection of contract type is determined by uncertainty in the definition of the project’s product and of the process to deliver it (Turner and Simister 2001). The efficacy of different contract types should be analyzed through a three dimensional schema: rewards, risk, and safeguard (Turner 2004). During the contract agreement stage, in dealing between client and contractor it is necessary to discuss and negotiate possible risk factors, risk and rewards strategies, and safeguards. Risk factor varies with the type of project and agreement, but severity and sharing patterns differ within the project contract agreement patterns. Usually the contract patterns are (i) fixed price contract, (ii) cost plus contracts, (iii) unit price contracts and (iv) time and material type contracts. Detailed description of different types of contracts can be obtained from Turner and Simister (2001) and Federal Acquisition Regulations website (http://www.arnet.gov/far/).

Depending upon involved degrees of uncertainties, contracts are varied with various incentives or profit structures. As uncertainties or risks increase for clients, contracts like firm fixed price (FFP) with economic price adjustment (FP/EPA), fixed price incentive (FPI), cost-plus- incentive-fee (CPIF), cost-plus-award-fee (CPAF), and cost-plus-fixed-fees (CAFF) type must be used to share the risks between the two parties. But there is no real theory of project or contract organization that says why this should be so (Turner and Simister 2001). In Figure 1, we show different categories of contracts for various degrees of client and contractor’s cost risks.

<table>
<thead>
<tr>
<th>Client’s cost risk</th>
<th>FFP</th>
<th>FP/EPA</th>
<th>FPI</th>
<th>CPIF</th>
<th>CPAF</th>
<th>CPFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contractor’s cost risk</td>
<td>Low</td>
<td>High</td>
<td>Low</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

Figure 1: Contract techniques and risk

Basically, risk is an undesirable phenomenon which grows from uncertainty. We characterize risk as a variation from a target value or a predicted value, which is seriously followed in project contracts. Risk on a project and choices of contract form are interlinked (Turner and Cochrane 1993). In terms of risk sharing pattern Firm Fixed Price (FFP) and Cost-plus-fixed-fee (CPFF) are the two extreme categories of contracts. FFP fixes the total paying price of the client at the time of agreement. The FFP is rigid as nothing can be changed after the agreement and the contractor shares all the uncertainties after the agreement. At the time of awarding contract the FFP system fixes the project fees considering all project costs, benefits and risks. The system is extremely inflexible and there is no provision of future change. From the client’s point of view the advantages of using FFP are many, for example the total payment does not change and there is no influence of uncertainty at time of the project operation. For contractors, profit may increase due to the cost savings provided customers are satisfied and quality is maintained. The drawbacks for the clients are the difficulties in preparing an accurate assessment of cost. Contractors alone have to handle any uncertainty during project running time i.e. no extra payment for any hidden risks and the possible influence of interest rates and currency exchange rates. Conversely, in CPFF the total cost risks are borne by the client. The client reimburses the contractor for all the audited costs of the project and pays an additional fee and hence the total price of the contract depends upon contract cost. Contract fee is not changed unless there is a change in work or terms of the contract. From the client’s perspective, cost plus contracts have the obvious disadvantage of managing and controlling the costs strictly, since final costs are not guaranteed (Jorgenson and Wallace, 2001).

Modern project management is a concept of teamwork, partnership and good communication among different stakeholders. The traditional win-lose situation is becoming obsolete and these days it is important to consider a win-win strategy through partnership. The failures of considering the incentives in FFP and CPFF contracts have led to greater consideration of incentive contracts. Incentive contracts are not new and have been utilized for years in private enterprise construction projects (Meinhart and Delionback 1968). According to Müller and Turn (2005),

1973
ideal form of contract is an alliance or partnership between the owner and the contractor. Compared to the two extremes (FFP and CPFF), incentive contracts are significantly unique in forming an alliance and in sharing risk and benefits between the client and the contractor. Introduction of an agreeable incentive system compresses the project risk and improves the performance level.

2.2 FPI Contract

FPI and CPIF are the two well-known types of cost incentive contracts and their relative positions compared to others can be seen from Figure 1. FPI contract is a fixed-price contract where price is adjustable by a provision that adjusts profit according to a formula based on the relationship of final negotiated actual cost to target cost (FAR 2005). From Figure 2, we can see that if the final actual costs \( A_c \) are less than the target costs \( T_c \), then the final profit for contractor is more than the target profit \( T_p - T_c \). Conversely, when \( A_c \) is more than the \( T_c \) (cost overrun), then final profit will be less than the target profit \( T_p - T_c \), or may even show a net loss. In case of FPI, the final profit plus actual cost is less than or equal to the ceiling price \( C_p \). If \( A_c \) is more than the \( C_p \), more loss would result for the contractor.

In Figure 2 we show the relationship between client’s profit and cost for FPI. For different prices we represent FFP and FPI risk points. It can be seen that profit varies inversely with cost. In FFP the maximum amount of payment to the contractor is target price \( T_p \). However in FPI, ceiling price \( C_p \) is the maximum amount that can be paid to the contractor. Therefore, the ceiling price is a crucial point in the negotiation of an FPI contract. The contractor sees \( C_p \) as his loss point and takes every step available to him to negotiate as high a ceiling price as possible (Rowland 1967). Before contractor’s expenses reaches to the \( C_p \) there is another point that is known as ‘point of total assumption’ or ceiling cost at which client stops sharing risk, and above ceiling cost contractors carry all the risks and the contract becomes FFP.

![Figure 2: Relationship between client’s profit and cost](image)

3. Proposed FPI Price Model

There is a broad division of game theory into two approaches: the cooperative and non-cooperative approach. These are two different approaches of looking at the same problem. The non-cooperative game is strategy oriented. Cooperative game theory, on the other hand directly looks at the set of possible outcomes, studies what the players can achieve, what coalitions will form, how the coalitions that do form divide the outcome, and whether the outcomes are stable and robust (Nagarajan and Sosic 2008). Our focus in this paper is to use cooperative game theory in FPI for risk sharing negotiation in a win-win situation.

We calculate the profit function \( GP_A \) and \( GP_B \) respectively for the client and the contractor. The anticipated profit and risk sharing agreements between the parties are considered in the profit function. For the client, we assume that there will be a gross profit \( M \) before adjusting risks, project actual cost \( A_c \) and contractor’s payment. For contractor, there are two functional elements in the profit functions and these are the target profit \( T_p - T_c \) and the contractor share of the cost risks \( T_c - A_c \). In the following equations we show the profit functions for client and contractor.
Client’s profit, \( GP_a = u_a = M + (1 - \alpha)(T_c - A_c) - A_c - GP_b \) \hspace{1cm} (1)

Contractor’s profit, \( GP_p = u_b = (T_p - T_c) + \alpha(T_c - A_c) \) \hspace{1cm} (2)

Here,

- \( M \) is the client gross profit from the project
- \( T_p \) is target price, \( T_c \) is target cost or budget, \( A_c \) is actual cost
- \( C_c \) is ceiling cost, \( p_C \) is ceiling price
- \( \alpha \) is the contractor’s risk sharing ratio i.e. the amount of profit the contractor gets for each dollar of actual cost saved or expended and \( 0 \leq \alpha \leq 1 \).

If we substitute \( GP_b \) in equation 1, then the client’s linear profit function will be as follows:

\[ GP_a = M + 2(1 - \alpha)(T_c - A_c) - T_p \] \hspace{1cm} (3)

In the proposed profit function computation, FPI policy differs from FFP in the following two points. Firstly, the difference is with the incentive clause for the reduction of the contractor risk. In FPI, the client and the contractor share the difference of actual cost \( A_c \) and target cost \( T_c \) in a fixed ratio. If \( A_c \leq T_c \), then there are contractors’ profits and if \( A_c > T_c \), then there will be a loss. Secondly, in case of if \( A_c > T_c \), there will be additional loss sharing, clients’ total payment will be more and hence the risk for client will be larger than traditional FFP. For sharing the risk between the contractor and client, it is important to have a suitable method of assigning an appropriate \( T_p \) in contrast with \( T_c \).

In this article our objective is to find an optimal \( T_p \) to resolve the risk-sharing problem. We consider that the risk-sharing problem should be at the consent of both the parties and for optimal target price determination use of cooperative game theory is a promising approach. The optimal contract is defined in principal-agent literature to be the one that minimizes the expected price paid by the principal (client) to the agent (contractor) and that identifies optimal incentive structure.

Considering risk sharing between the client and the contractor principally supports a mutual understanding and a win-win situation in the FPI contract. With that aim our purpose is to develop a cooperative game theory based FPI price fixing model so that both parties will benefit by maximizing profit in projects. In our analysis we compare FPI with FFP as they have the same origin.

In FPI, it is expected that both contractor and client should have an interest to maximize gain from the contract. Considering individual objective and overall goal, we consider the problem as a cooperative game between the two parties. We presume that both contractor and client will cooperate to maximize their minimum profit, therefore their bargaining benchmark will be maximizing the minimum outcome of the profit i.e. minimax value of the profit.

The bargaining benchmark for both parties is at the bottom line of the profit and that is the bargaining starting point. In order to maximize biasness surplus profit from the bottom line, it is sufficient to maximize the surplus profit amount \( F \) from the benchmark point. We consider \( u_A \) and \( u_B \) as final compromise profit points, and \( u_A^0 \) and \( u_B^0 \) as benchmark profit points for the bargaining for clients and contractors respectively. The proposed surplus profit function is shown in equation 4.

\[ \text{Max. } F = (u_A - u_A^0)(u_B - u_B^0) = [M + (1 - \alpha)(T_c - A_c) - A_c - u_B^0][u_B - u_B^0] \] \hspace{1cm} (4)

After assigning the benchmark profit points in equation 4, we maximize the profit function \( \frac{\partial F}{\partial u_B} = 0 \) and obtain the optimum profit values \( u_A^* \) and \( u_B^* \) for the client’s and contractor’s respectively. Summary of the equation for \( \frac{\partial F}{\partial u_B} = 0 \) is shown below. Derivations and calculations are detailed in the Appendix.

\[ \frac{\partial F}{\partial u_B} = M + (1 - \alpha)(T_c - A_c) - A_c - u_A^0 + 2u_B + u_B^0 \]

Further we analyze profit functions for different risk factors for the following two cases.

**Case 1:** When actual project cost is lower than the target cost \( A_c \leq T_c \)
For this case, $\alpha = 0$, i.e. no risk is shared by the contractor. If we consider both the target price and ceiling price as equal ($T_p = C_p$) then the maxi-min value of $GP_A$ and $GP_B$ will be fulfilled and have the following result for benchmark profits at bargaining:

Maximin. $GP_A = M + 2(T_c - A_c) - C_p = u_A^0$

Maximin. $GP_B = C_p - T_c = u_B^0$

Substituting the values of $u_A^0$ and $u_B^0$ in equation 4 we can obtain optimum profit values:

For client, $u_A^* = M + (2 - 0.5\alpha)(T_c - A_c) - C_p$

For contractor, $u_B^* = (C_p - T_c) - 0.5\alpha(T_c - A_c)$

To obtain target price $T_{p1}$, we substitute $GP_B = u_B = u_B^*$, then $T_{p1}$ can be calculated as the following relation:

$$T_{p1} = C_p - 1.5\alpha(T_c - A_c)$$  (5)

Case 2: When actual project cost is higher than the target cost ($A_c > T_c$)

In this case we consider that all risks are shared by the contractor ($\alpha = 1$). If both the target price and ceiling price are equal ($T_p = C_p$) then the maxi-min value of $GP_A$ and $GP_B$ will be as follows:

Maximin. $GP_A = M - C_p = u_A^0$

Maximin. $GP_B = C_p - A_c = u_B^0$

Substituting the values of $u_A^0$ and $u_B^0$ in equation 4 we can obtain optimum profit values:

$u_A^* = M + 0.5(1-\alpha)(T_c - A_c) - C_p$

$u_B^* = (C_p - A_c) + 0.5(1-\alpha)(T_c - A_c)$

Like case 1, to determine target price for the contractor $T_{p2}$, we substitute $GP_B = u_B = u_B^*$ and calculate $T_{p2}$ as the following relation:

$$T_{p2} = C_p + 1.5(1-\alpha)(T_c - A_c)$$  (6)

**Expected Value of Target Price**

We choose two different cases of risk sharing and determine two different target prices. As the actual cost of the project is unknown at the bargaining point, the emphasis is on determining the expected target price of the project. To calculate expected target price $E(T_p)$, we set an 80% probability for $A_c \leq T_c$ at and a 20% probability for $A_c > T_c$.

On the basis of the set probabilities, the resulting expected project target price can be obtained from the following expression:

$$E(T_p) = 0.8T_{p1} + 0.2T_{p2}$$

$$= 1.5\alpha T_c - 1.5\alpha T_1 - 0.3A_c + 0.3T_c + C_p = C_p - 1.5\alpha(T_c - A_c) + 0.3(T_c - A_c)$$

At the bargaining point the value of actual cost $A_c$ is unknown. We therefore assume that at that point actual cost and target cost are equal, i.e. $A_c = T_c$. Considering this value of actual cost, from the above equation we can obtain the optimal FPI project value or expected target price, $E(T_p) = C_p$. This implies that at the bargaining point optimal FPI project value is the ceiling price of the contract. Further, we contrast the payment value of FFP with FPI. We premise that at the bargaining point the client’s risk is proportional to the total payment value to the contractor. Simplifying the assumption the risk of FFP and FPI system can be determined as follows:

$$\text{Client’s Payment(FFP)} = T_p \quad \text{and} \quad \text{Client’s Payment(FPI)} = A_c + GP_B$$  (7)

**Numerical Example**

We explain the model and determine different payment and profit values for FFP and FPI with a numerical example for case 1 and case 2.
Example: If we take a hypothetical example of a project with $T_p = $10 millions, risk sharing ratio for contractor 30%, $C_p = $12 millions. We assume for FFP target price $T_p = $11 millions

**Case 1:** If $A_c = $9 millions, $T_{p1} = 12 - 1.5 (0.3) (10 - 9) = $11.5 millions (from equation 5)
Payment (FFP), $T_p = $11 millions; Contractor’s profit = $2 millions
Contractor’s profit (FPI) = $(11.5 - 10) + 0.3(10 - 9) = $1.85 millions (from equation 2)
Client’s final payment value (FPI) = $9 + 1.85 (from equation 7) = $10.85M, savings for the client $(11.5-10.85)$ or $0.65 million. The contractor earns $1.85 millions that is less than target profit $(T_{p1} - A_c)$ or $2.5 millions. If $A_c < T_p$: for FPI, the contractor will save some cost and will receive some cost saving benefits. Similarly, the client will save some cost that will offset client’s total payment value. On the other hand for FFP, there is no risk sharing at all.

**Case 2:** If $A_c = $11 millions, $T_{p2} = 12 + 1.5(0.7) (10-11) = $10.95 millions (from equation 6)
For FFP, final payment = $11 millions = $A_c and contractor’s profit = 0
For contractor’s profit (FPI) = $(10.95 - 10) + 0.3(10-11) = $0.65 millions (from equation 2)
Client’s final payment value (FPI) = $11 + 0.65 = 11.65 (from equation 7) > $T_{p2}$
Final savings for the client $(10.95 - 11.65)$ million or -$0.7 million. The contractor earns a $0.65 millions that is more than target profit $(T_{p2} - A_c)$. For FPI, the contractor will make less profit compared to case 1. Similarly, client will share more risk and consequently total payment will be higher than case 1.

From the comparison between client’s payment and contractor’s payment it can be seen that with the increase in actual cost, contractor’s profit is decreasing and client’s payment is increasing. Compared to FFP, in FPI client shares more risk when actual cost is greater than target cost. In terms of profit, when actual cost is lower than the target cost, FPI shares more benefits to both.

4. Conclusion

FPI contract is applicable when FFP contracts are inappropriate, when it is desirable for the contractor to assume some cost responsibility, and when a firm target and a formula can be negotiated at the beginning (FAR 2005). We propose a model that resolves the contract dealing of FPI and calculates the bargaining value of target project price $T_p$. We consider different risk factors in FPI contract and determine the corresponding target prices. With a numerical example we show the expected value of the project. We further find out the initial risk of the client and compare with the FFP.

The model can be extended in many ways as the developed optimal formula is not applicable when the actual cost is more than the ceiling price. Further development of this model could be done at the point of bargaining ($A_c = T_p$) for FPI. On the other hand many other factors need to be considered to adapt FPI these days, especially for the cases where the client’s risk increases by adopting FPI. The question arises whether the client will adopt this strategy with good grace. The client needs to understand that the ultimate cost can be reduced while attaining the expected quality by adopting the FPI.

References


Appendix:

Surplus profit, $F = (u_A - u_A^0)(u_B - u_B^0)$

$$= [M + (1 - \alpha)(T_c - A_c) - A_c - u_B - u_B^0][u_B - u_B^0]$$

$$= M(u_B - u_B^0) + (1 - \alpha) (T_c - A_c)(u_B - u_B^0) - A_c(u_B - u_B^0) - u_A^0(u_B - u_B^0) - u_A^2 + u_Bu_B^0$$

$$\frac{\partial F}{\partial u_B} = M + (1 - \alpha)(T_c - A_c) - A_c - 2u_B + u_B^0$$

$$\frac{\partial F}{\partial u_B} = 0$$

Case 1: $A_c \leq T_c$

If, Maximin. $GP_A = M + 2(T_c - A_c) - C_p = u_A^0$ and Maximin. $GP_B = C_p - T_c = u_B^0$

Then, $M + (1 - \alpha)(T_c - A_c) - A_c - 2u_B + u_B^0 = 0$

$$\Rightarrow M + (1 - \alpha)(T_c - A_c) - A_c - M - 2(T_c - A_c) + C_p - 2u_B + C_p - T_c = 0$$

$$\Rightarrow u_B = C_p - T_c - 0.5(T_c - A_c)$$

$$\Rightarrow u_B^* = (C_p - T_c - 0.5)(T_c - A_c)$$

$$u_A = M + (1 - \alpha)(T_c - A_c) - A_c - u_B^*$$

$$u_A^* = M + (2 - \alpha/2)(T_c - A_c) - C_p$$

Case 2: $A_c \geq T_c$

If, Maximin. $GP_A = M - C_p = u_A^0$ and Maximin. $GP_B = C_p - A_c = u_B^0$

Then, $\frac{\partial F}{\partial u_B} = 0$

$$\Rightarrow M + (1 - \alpha)(T_c - A_c) - 2u_B + u_B^0 - A_c - u_A^0 = 0$$
\[ M^+ (1-\alpha)(T_c - A_c) - 2u_B + (C_p - A_c) + A_c - (M - C_p) = 0 \]

\[ 2u_B = (1-\alpha)(T_c - A_c) + 2(C_p - A_c) \]

\[ u_B^* = (C_p - A_c) + 0.5(1-\alpha)(T_c - A_c) \]

In equation 1, if we substitute \( u_B \) and \( u_A \) respectively by \( u_A^* \) and \( u_B^* \), then

\[ u_A^* = M + (1-\alpha)(T_c - A_c) - A_c - u_B^* \]

\[ u_A^* = M + 0.5(1-\alpha)(T_c - A_c) - C_p \]